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ABSTRACT

This paper presents a method for estimating certain characteristics of test items which are designed to measure ability, or knowledge, in a particular area. Under the assumption that ability parameters are sampled from a normal distribution, the EM algorithm is used to derive maximum likelihood estimates to item parameters of the two-parameter logistic item response curves. The observed information matrix is used to approximate the covariances of these estimates. Responses to a questionnaire on general arthritis knowledge are used to illustrate the procedure and simulated data are used to compare the actual versus estimated item parameters. A computational note is included to facilitate the extensive numerical work required to implement the procedure. (Author)

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This paper presents a method for estimating certain characteristics of test items which are designed to measure ability, or knowledge, in a particular area. Under the assumption that ability parameters are sampled from a normal distribution, the EM algorithm is used to derive maximum likelihood estimates of item parameters of the two-parameter logistic item response curves. The observed information matrix is used to approximate the covariances of these estimates.

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ESTIMATION OF TWO-PARAMETER LOGISTIC ITEM RESPONSE CURVES

by

Robert K. Tsutakawa
University of Missouri-Columbia

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INTRODUCTION

We consider dichotomous responses by subjects to a set of test items, when the responses are assumed to follow the two parameter logistic model. Under the assumption that ability parameters are randomly sampled from a normal distribution with unknown parameters, empirical Bayes types estimates of ability and item parameters were proposed and illustrated for the one-parameter logistic (Rasch) model by Rigdon and Tsutakawa (1983). Here we will consider one of these estimators, namely MLF, where the extension to multiparameter item response models is straightforward and asymptotic methods are available from the theory of maximum likelihood estimation.

A standard procedure for estimating both ability and item parameters in the absence of prior distributions is maximum likelihood (cf. Lord 1980). Maximum likelihood estimation when the ability parameters have a common prior distribution have been studied by a number of authors, including, among others, Bock and Lieberman (1970), Andersen and Madsen (1977), Sanathanan and Blumenthal (1978), and Bock and Aitkin (1981). The latter two papers use different forms of the EM algorithm (Dempster, Laird, and Rubin 1977) to greatly simplify what previously appeared to be an insurmountable numerical problem.

This paper shows that by using the extended form of the EM algorithm, which does not require the presence of sufficient statistics, the computational difficulty is reduced to solving a series of 2 equations with 2 unknown, corresponding to the 2 parameters of the logistic curve. Although the equations must be numerically solved by some iterative method, the task is a vast simplification over methods where all parameters are found simultaneously. Under the

assumption that the ability parameters are independent and identically distributed, the classical maximum likelihood theory applies and the inverse of the observed information matrix may be used as an approximate covariance matrix of the estimated item parameters. The successful use of our method requires a fair amount of computation. Most of the computational burden is in accurately evaluating a large number of single intergals, which are basically expectations with respect to posterior distributions of abilities.

After describing our model and assumptions, we discuss the implementation of the EM algorithm to our problem. The equations at each iteration of the algorithm is then related to the likelihood equations. Responses to a questionnarie on arthritis knowledge will then be used to illustrate our method and simulations will be used to illustrate the reliability of the observed information matrix for assessing the errors in the item parameter estimates. Expressions of derivatives needed to carry out the computation are summarized in the last section.

ITEM RESPONSE MODEL

Consider n subjects responding to k test items designed to measure the ability of the subjects in a particular area. Let $\underset{\sim}{Y}$ be an $n \times k$ matrix of binary responses where $Y_{ij} = 1$ or 0 accordingly as the ith subject's response to item j is correct or incorrect, $i = 1,\cdots,n$ and $j = 1,\cdots,k$. A general framework in which to study such item responses is to consider some probability model,

$$p_{ij} = P(Y_{ij} = 1|\theta_i,\xi_j), \tag{2.1}$$

depending on a real valued ability parameter of the ith subject, θ_i, and a real or vector valued item parameter of the jth item, $\underset{\sim}{\xi}_j$. Given $\underset{\sim}{\theta}^T = (\theta_1,\cdots,\theta_n)$ and $\underset{\sim}{\xi}^T = (\underset{\sim}{\xi}_1^T, \cdots,\underset{\sim}{\xi}_k^T)$, where T denotes transpose, conditional independence among the responses in $\underset{\sim}{Y}$ is assumed, so that the joint probability of $\underset{\sim}{Y} = \underset{\sim}{y}$, given $(\underset{\sim}{\theta}, \underset{\sim}{\xi})$ is

$$P(\underset{\sim}{y}|\underset{\sim}{\theta},\underset{\sim}{\xi}) = \prod_{i=1}^{n}\cdot\prod_{j=1}^{k} p_{ij}^{y_{ij}} (1 - p_{ij})^{1-y_{ij}}. \qquad (2.2)$$

We further assume that $\underset{\sim}{\theta}$ may be treated as a random sample from some prior distribution with pdf $p(\theta|\underset{\sim}{\lambda})$ having parameter $\underset{\sim}{\lambda}$.

In this paper we shall be discussing the estimation of $\underset{\sim}{\xi}$ in the special case of the two-parameter logistic model, defined by

$$P(y_{ij}|\theta_i,\alpha_j,\beta_j) = \frac{e^{y_{ij}\,\alpha_j(\theta_i - \beta_j)}}{1 + e^{\alpha_j(\theta_i - \beta_j)}}, \qquad (2.3)$$

$y_{ij} = 0,1$, where $\underset{\sim}{\xi}_j$ is now the vector (α_j,β_j) with $0 < \alpha_j < \infty$ and $-\infty < \beta_j < \infty$. For the prior of θ, we consider the normal distribution with mean μ and variance σ^2 so that $\underset{\sim}{\lambda} = (\mu,\sigma^2)$. However, due to the nonuniqueness of this parametrization we shall add the constraints $\mu=0$ and $\sigma^2=1$. We will thus assume that the prior pdf of θ is

$$p(\theta) = (2\pi)^{-1/2} \exp(-\theta^2/2), \qquad (2.4)$$

$-\infty < \theta < \infty$. It can be shown (Tsutakawa 1982), that there is an equivalence between this parametrization and the more familiar one

where the constraints are

$$\sum_{j=1}^{k} \beta_j = 0 \quad \text{and} \quad \prod_{j=1}^{k} \alpha_j = 1,$$

with no restriction on (μ,σ^2).

For the above model, the joint distribution of $(\underset{\sim}{Y}_i^T,\theta_i)$, where $\underset{\sim}{Y}_i^T = (Y_{i1},\ldots,Y_{ik})$, is given by

$$f(\underset{\sim}{y}_i^T,\theta_i|\underset{\sim}{\xi}) = p(\theta_i) \prod_{j=1}^{k} P(y_{ij}|\theta_i,\alpha_j,\beta_j) \tag{2.5}$$

and the joint distribution of $(\underset{\sim}{Y},\underset{\sim}{\theta})$ by

$$f(\underset{\sim}{y},\underset{\sim}{\theta}|\underset{\sim}{\xi}) = \prod_{j=1}^{n} f(y_i^T,\theta_i|\underset{\sim}{\xi}). \tag{2.6}$$

Thus the marginal probability of response $\underset{\sim}{y}_i$ for the ith subject is

$$P(\underset{\sim}{y}_i^T|\underset{\sim}{\xi}) = \int f(y_i^T,\theta_i|\underset{\sim}{\xi})d\theta_i \tag{2.7}$$

and for all subjects

$$P(\underset{\sim}{y}|\underset{\sim}{\xi}) = \prod_{i=1}^{n} P(\underset{\sim}{y}_i^T|\underset{\sim}{\xi}). \tag{2.8}$$

The loglikelihood function of $\underset{\sim}{\xi}^T = \{(\alpha_1,\beta_1),\ldots,(\alpha_k,\beta_k)\}$ is thus given by

$$L(\underset{\sim}{\xi}) = \sum_{i=1}^{n} \log \int p(\theta_i) \prod_{j=1}^{k} P(y_{ij}|\theta_i,\alpha_j,\beta_j)d\theta_i. \tag{2.9}$$

PARAMETER ESTIMATION VIA THE EM ALGORITHM

In this section we outline the use of EM algorithm for obtaining $\hat{\underset{\sim}{\xi}}$, the maximum likelihood estimate of $\underset{\sim}{\xi}$, when the loglikelihood function is given by (2.9). Basically the algorithm consists of iteratively deriving the value of $\underset{\sim}{\xi}$ which maximizes the posterior expectation

$$T(\underset{\sim}{\xi}) = E\{\log f(\underset{\sim}{y},\underset{\sim}{\theta}|\underset{\sim}{\xi})|\ \underset{\sim}{y},\underset{\sim}{\xi}^{\circ}\}, \tag{3.1}$$

given $\underset{\sim}{y}$ and a provisional estimate $\underset{\sim}{\xi}^{\circ}$, where $\underset{\sim}{\xi}^{\circ}$ is initially some approximation to $\hat{\underset{\sim}{\xi}}$ and is replaced at the end of each iteration by the maximizing value of $\underset{\sim}{\xi}$.

For the two-parameter logistic model (2.3), each iteration reduces to solving the equations,

$$\sum_{i=1}^{n} y_{ij} = \sum_{i=1}^{n} \int \frac{p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi}^{\circ})}{1+\exp\{-\alpha_j(\theta_i-\beta_i)\}}\, d\theta_i\,, \tag{3.2}$$

$$\sum_{i=1}^{n} y_{ij} \int \theta_i p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi}^{\circ})\, d\theta_i = \sum_{i=1}^{n} \int \frac{\theta_i p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi}^{\circ})}{1+\exp\{-\alpha_j(\theta_i-\beta_i)\}}\, d\theta_i,$$

for (α_j,β_j), separately for $j = 1,\ldots,k$, where

$$\underset{\sim}{\xi}^{\circ} = \{(\alpha_1^{\circ},\beta_1^{\circ}),\ldots,(\alpha_k^{\circ},\beta_k^{\circ})\}^T, \quad \text{and}$$

$$p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi}^{\circ}) = \frac{f(\underset{\sim}{y}_i,\theta_i|\underset{\sim}{\xi}^{\circ})}{P(\underset{\sim}{y}_i|\underset{\sim}{\xi}^{\circ})}\,, \tag{3.3}$$

the posterior pdf of θ_i, given $\underset{\sim}{\xi} = \underset{\sim}{\xi}^\circ$ and observation $\underset{\sim}{y}_i$, i=1,...,n. The solution at the end of each iteration is used as the value of $\underset{\sim}{\xi}^\circ$ in the next iteration, and the process continues till convergence is attained.

Since the solution to the equation (3.2) has no simple expression it must be derived numerically by iterative methods such as the one by Marquardt (1963). Moreover, the integrals must be evaluated by numerical techniques such as Gauss-Hermite. The expressions for the derivative needed for the implementation these methods are summarized in the last section.

It is instructive to compare equations (3.2) with the likelihood equation which $\hat{\underset{\sim}{\xi}}$ satisfies. The likelihood equations, which are obtained by setting the first partial derivatives of (2.9) with respect to α_j and β_j equal to zero, are

$$\sum_{i=1}^{n} y_{ij} = \sum_{i=1}^{n} \int \frac{p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi})}{1 + \exp\{-\alpha_j(\theta_i-\beta_j)\}}\, d\theta_i \,,$$

$$\sum_{i=1}^{n} y_{ij} \int \theta_i p(\theta_i|y_i,\underset{\sim}{\xi})d\theta_i = \sum_{i=1}^{n} \int \frac{\theta_i p(\theta_i|\underset{\sim}{y}_i,\underset{\sim}{\xi})}{1 + \exp\{-\alpha_j(\theta_i-\beta_j)\}}\, d\theta_i, \tag{3.4}$$

j=1,...,k, where $\underset{\sim}{\xi} = \{(\alpha_1,\beta_1),\ldots,(\alpha_k,\beta_k)\}$ The only difference between the two systems of equations (3.2) and (3.4) is in the role $\underset{\sim}{\xi}$ and $\underset{\sim}{\xi}^\circ$. In (3.2) $\underset{\sim}{\xi}^\circ$ is a fixed known value at each iteration, whereas in (3.4) $\underset{\sim}{\xi}$ is an unknown to be solved. Solving (3.4) directly requires simultaneously finding 2k unknowns in 2k equations.

On the other hand, each iteration of the EM algorithm only requires finding a series of 2 unknowns in 2 equations. Note that once convergence is attained using the EM algorithm, the successive values of $\underset{\sim}{\xi}^{\circ}$ remain unchanged and satisfy the likelihood equations (3.4).

ERRORS IN PARAMETER ESTIMATES

Since the vector valued observations $\underset{\sim}{Y}_1, \underset{\sim}{Y}_2, \ldots, \underset{\sim}{Y}_n$ are independent with a common distribution (2.7), we appeal to the asymptotic properties of the maximum likelihood estimator to assess the reliability of $\hat{\underset{\sim}{\xi}}$. For large samples, $\hat{\underset{\sim}{\xi}}$ is approximately normal with mean $\underset{\sim}{\xi}$ and covariance $I^{-1}(\underset{\sim}{\xi})$, the reciprocal of the Fisher information matrix $I(\underset{\sim}{\xi})$. In practice $\underset{\sim}{\xi}$ is unknown and $I(\underset{\sim}{\xi})$ is difficult to compute. We therefore propose estimating $I(\underset{\sim}{\xi})$ by the observed information matrix $\mathcal{I}(\hat{\underset{\sim}{\xi}})$, defined as the negative second derivative matrix of the loglikelihood function (2.9). Justifications for this approximation are given in Efron and Hinkley (1978). Expressions of $\mathcal{I}(\underset{\sim}{\xi})$ suitable for numerical computation are summarized below under computational notes. Numerical examples of this approximation are shown in the next two sections.

APPLICATION TO ARTHRITIS KNOWLEDGE

We illustrate our method using responses from hospital patients to a 50 item questionnaire on general information regarding arthritis. The questionnarie was completed by n = 167 subjects. Of the original 50 items, 3 were deleted since their parameters could not be estimated, and we thus used the remaining k = 47 items.

Estimates of (α_j, β_j) and there covariances are summarized in Table 1.

Insert Table 1 about here

SIMULATION

To obtain some indication of performance, our method was applied to a randomly generated data set, $\underset{\sim}{Y}$, using n = 200 ability parameters randomly selected from a standard normal distribution and k = 50 pairs of item parameters (α_j, β_j) whose values are in the range typically encountered in pratice. Figures 1, 2 and 3 present plots of the estimated versus simulated values of α, β, and θ. The estimates for θ are posterior means obtained from (3.3) with $\underset{\sim}{\xi}^{\circ} = \hat{\underset{\sim}{\xi}}$, as in Rigdon and Tsutakawa (1983). We note a fairly high correlation of .988 and .974 for β and θ, respectively, but a somewhat lower one, .881, for α. The estimated covariances for $(\hat{\alpha}, \hat{\beta})$ are summarized in Table 2 together with standardized errors $(\hat{\alpha} - \alpha)/SD(\hat{\alpha})$ and $(\hat{\beta} - \beta)/SD(\hat{\beta})$, where SD's are square roots of the variances approximated by $I^{-1}(\hat{\underset{\sim}{\xi}})$. The simulation was repeated using n = 400 and k = 25, with similar results on the standardized errors. Although these results appear quite plausible, until we obtain more experience using $I(\hat{\underset{\sim}{\xi}})$ the method should be used with caution for data sets of smaller sizes or when the model may not be appropriate.

Insert Figure 1,2, & 3 and Table 2 about here

COMPUTATIONAL NOTES

We now summarize expressions for the first two derivatives of the loglikelihood function (2.9) and the posterior expectation (3.1), which may be used to evaluate $I(\hat{\underset{\sim}{\xi}})$ and to maximize $T(\underset{\sim}{\xi})$, respectively.

For each i = 1,...,n and u = 1,...,k let

$$g_1(i,u,\theta) = \frac{\partial P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \alpha_u} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$g_2(i,u,\theta) = \frac{\partial P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \beta_u} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{11}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \alpha_u^2} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{12}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \alpha_u \, \partial \beta_u} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$$h_{22}(i,u,\theta) = \frac{\partial^2 P(y_{iu}|\theta,\alpha_u,\beta_u)}{\partial \beta_u^2} \Big/ P(y_{iu}|\theta,\alpha_u,\beta_u),$$

$\phi_{\theta u} = \{1 + \exp[-\alpha_u(\theta - \beta_u)]\}^{-1}$, and $\psi_{\theta u} = 1 - \phi_{\theta u}$.

By taking derivatives it is readily shown that

$g_1(i,u,\theta) = (y_{iu} - \phi_{\theta u})(\theta - \beta_u)$,

$g_2(i,u,\theta) = -\alpha_u(y_{iu} - \phi_{\theta u})$,

$h_{11}(i,u,\theta) = (y_{iu} - \phi_{\theta u})(\psi_{\theta u} - \phi_{\theta u})(\theta - \beta_u)^2$,

$h_{12}(i,u,\theta) = (y_{iu} - \phi_{\theta u})[1 + \alpha_u(\psi_{\theta u} - \phi_{\theta u})(\theta - \beta_u)]$,

and

$h_{22}(i,u,\theta) = (y_{iu} - \phi_{\theta u})\,\alpha_u^2(\psi_{\theta u} - \phi_{\theta u})$.

Now define the following posterior expectations of the derivatives and their products by

$$\bar{g}_s(i,u) = \int g_s(i,u,\theta)p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi})d\theta, \tag{7.1}$$

$$\bar{h}_{st}(i,u) = \int h_{st}(i,u,\theta)p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi})d\theta, \tag{7.2}$$

$$\bar{d}_{st}(i,u,v) = \int g_s(i,u,\theta)g_t(i,v,\theta)p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi})d\theta, \tag{7.3}$$

$s,t = 1,2$, $u,v = 1,\ldots,k$, $u \neq v$, where $p(\theta|\underset{\sim}{y}_i,\underset{\sim}{\xi})$ is defined by (3.3).

Then the first and second derivatives of the loglikelihood function $L(\underset{\sim}{\xi})$, defined by (2.9), may be expressed by

$$\frac{\partial L(\underset{\sim}{\xi})}{\partial \alpha_u} = \sum_{i=1}^{n} \bar{g}_1(i,u), \tag{7.4}$$

$$\frac{\partial^2 L(\underset{\sim}{\xi})}{\partial \beta_u} = \sum_{i=1}^{n} \bar{g}_2(i,u), \tag{7.5}$$

$$\frac{\partial^2 L(\underset{\sim}{\xi})}{\partial \alpha_u \partial \alpha_v} = \begin{cases} \sum_{i=1}^{n} \{\bar{h}_{11}(i,u) - \bar{g}_1^2(i,u)\} & \text{if } u = v \\ \sum_{i=1}^{n} \{\bar{d}_{11}(i,u,v) - \bar{g}_1(i,u)\bar{g}_1(i,v)\} & \text{if } u \neq v, \end{cases} \tag{7.6}$$

$$\frac{\partial^2 L(\underset{\sim}{\xi})}{\partial \alpha_u \partial \beta_v} = \begin{cases} \sum_{i=1}^{n} \{\bar{h}_{12}(i,u) - \bar{g}_1(i,u)\bar{g}_2(i,u)\} & \text{if } u = v \\ \sum_{i=1}^{n} \{\bar{d}_{12}(i,u,v) - \bar{g}_1(i,u)\bar{g}_2(i,v)\} & \text{if } u \neq v, \end{cases} \tag{7.7}$$

$$\frac{\partial^2 L(\underset{\sim}{\xi})}{\partial \beta_u \partial \beta_v} = \begin{cases} \sum_{i=1}^{n} \{\bar{h}_{22}(i,u) - \bar{g}_2^2(i,u)\} & \text{if } u = v \\ \sum_{i=1}^{n} \{\bar{d}_{22}(i,u,v) - \bar{g}_2(i,u)\bar{g}_2(i,v)\} & \text{if } u \neq v, \end{cases} \tag{7.8}$$

for $u,v = 1,\ldots,k$.

For any pair $(\underset{\sim}{\xi}, \underset{\sim}{\xi}^\circ)$, define the posterior expectations of derivativies and their products,

$$\tilde{g}_s(i,u) = \int g_s(i,u,\theta)\ p(\theta|\underset{\sim}{y}_i, \underset{\sim}{\xi}^\circ)d\theta, \tag{7.9}$$

$$\tilde{h}_{st}(i,u) = \int h_{st}(i,u,\theta)\ p(\theta|\underset{\sim}{y}_i, \underset{\sim}{\xi}^\circ)d\theta, \tag{7.10}$$

$$\tilde{d}_{st}(i,u) = \int g_s(i,u,\theta)g_t(i,u,\theta)p(\theta|\underset{\sim}{y}_i, \underset{\sim}{\xi}^\circ)d\theta\ . \tag{7.11}$$

These integrals are indentical to (7.1) - (7.3) when $\underset{\sim}{\xi} = \underset{\sim}{\xi}^\circ$, but are generally not identical to them when $\underset{\sim}{\xi} \neq \underset{\sim}{\xi}^\circ$. The derivatives of $T(\underset{\sim}{\xi})$ may then be summarized by

$$\frac{\partial T(\underset{\sim}{\xi})}{\partial \alpha_u} = \sum_{i=1}^{n} \tilde{g}_1(i,u), \tag{7.12}$$

$$\frac{\partial T(\underset{\sim}{\xi})}{\partial \beta_u} = \sum_{i=1}^{n} \tilde{g}_2(i,u), \tag{7.13}$$

$$\frac{\partial^2 T(\underset{\sim}{\xi})}{\partial \alpha_u \partial \alpha_v} = \begin{cases} \sum_{i=1}^{n} \{\tilde{h}_{11}(i,u) - \tilde{d}_{11}(i,u)\} & \text{if } u = v \\ 0 & \text{if } u \neq v, \end{cases} \tag{7.14}$$

$$\frac{\partial^2 T(\underset{\sim}{\xi})}{\partial \alpha_u \partial \beta_v} = \begin{cases} \sum_{i=1}^{n} \{\tilde{h}_{12}(i,u) - \tilde{d}_{12}(i,u)\} & \text{if } u = v \\ 0 & \text{if } u \neq v, \end{cases} \tag{7.15}$$

$$\frac{\partial^2 T(\underset{\sim}{\xi})}{\partial \beta_u \partial \beta_v} = \begin{cases} \sum_{i=1}^{n} \{\tilde{h}_{22}(i,u) - \tilde{d}_{22}(i,u)\} & \text{if } u = v \\ 0 & \text{if } u \neq v, \end{cases} \tag{7.16}$$

$u,v = 1,\ldots,k.$

We note that the numerical solution to (3.2) and the evaluation of $I(\hat{\underset{\sim}{\xi}})$ are quite sensitive to the accuracy of the numerical approximation of integrals. A typical integral is a posterior expectation and, except for a constant factor, has the form

$$\int H(\theta)\, p(\theta) \prod_{j=1}^{k} P(y_{ij}|\theta,\alpha_j,\beta_j)\, d\theta, \tag{7.17}$$

where the function H varies from integral to integral. The missing constant factor is the reciprocal of this integral when $H(\theta) \equiv 1$. Since $p(\theta)$ is the standard normal pdf, the integral has the Mth oder Gauss-Hermite approximation

$$\pi^{-1/2} \sum_{\ell=1}^{M} H(\sqrt{2}\, x_\ell) \prod_{j=1}^{k} P(y_{ij} | \sqrt{2}\, x_\ell, \alpha_j, \beta_j) w_\ell \qquad (7.18)$$

where $(x_1,w_1),\ldots,(x_m,w_M)$ are the nodes and weights. (See Strout and Secrest 1966). Since the nodes are based on the prior of θ_i and not its posterior, there is some loss in accuracy when the posterior is displaced from the prior. To avoid this loss one can select the nodes around the posterior mean m_i using the posterior standard deviation c_i as the scale factor. To find m_i and c_i we first use (7.18) with $H(\theta) = \theta$ for m_i and $H(\theta) = (\theta - m_i)^2$ for c_i^2. Then the Gauss-Hermite approximation to (7.17) becomes

$$\sqrt{2}\, c_i \sum_{\ell=1}^{M} w_\ell\, H(m_i + \sqrt{2}\, c_i\, x_\ell)\, \exp(x_\ell^2)$$

$$\times\, p(m_i + \sqrt{2}\, c_i\, x_\ell) \prod_{j=1}^{k} P(y_{ij} | m_i + \sqrt{2}\, c_i\, x_\ell, \alpha_j, \beta_j).$$

FIGURE 1

ESTIMATED VS. ACTUAL ALPHA
FROM SIMULATION WITH
N=200 AND K=50

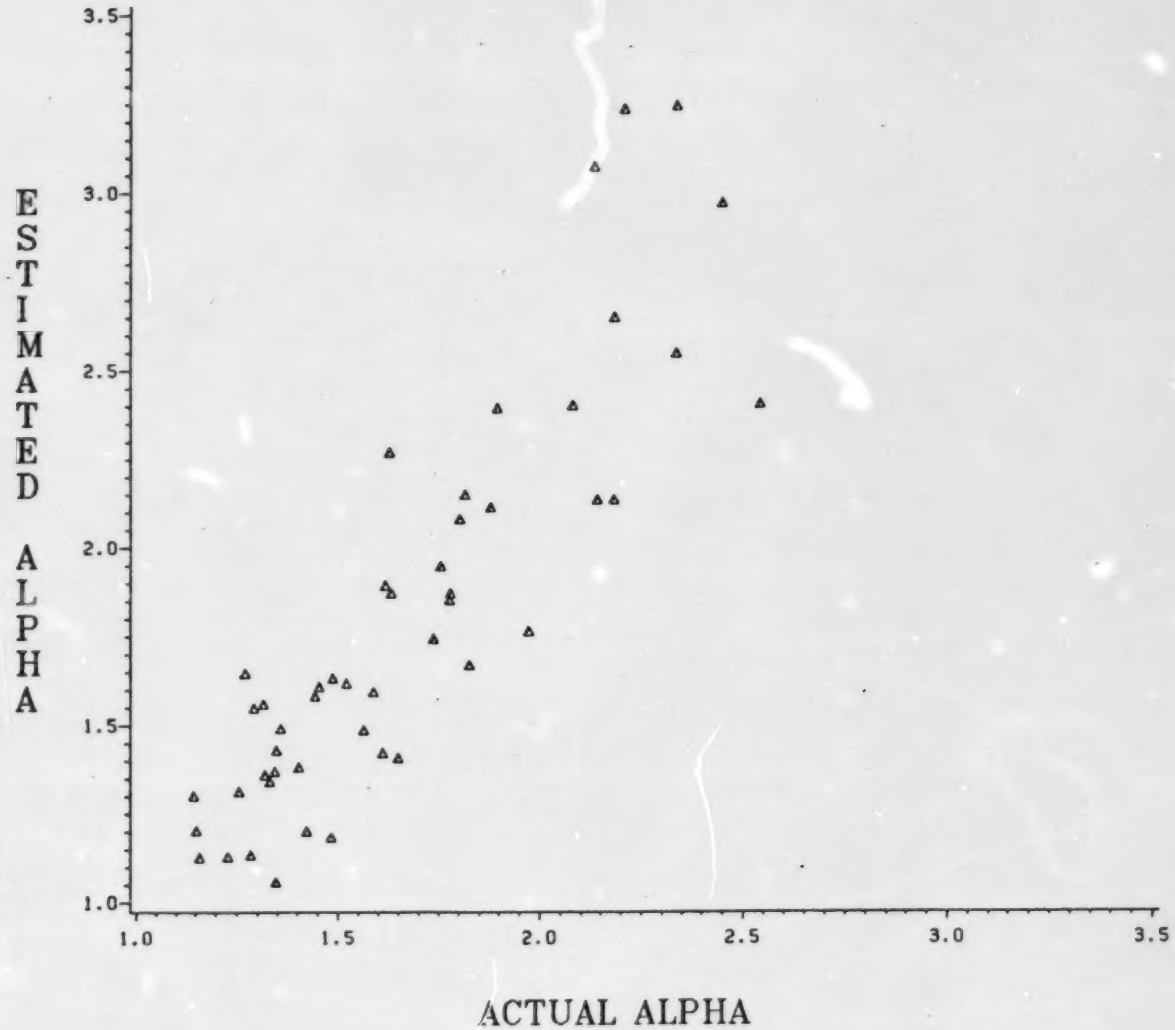

FIGURE 2

ESTIMATED VS. ACTUAL BETA
FROM SIMULATION WITH
N=200 AND K=50

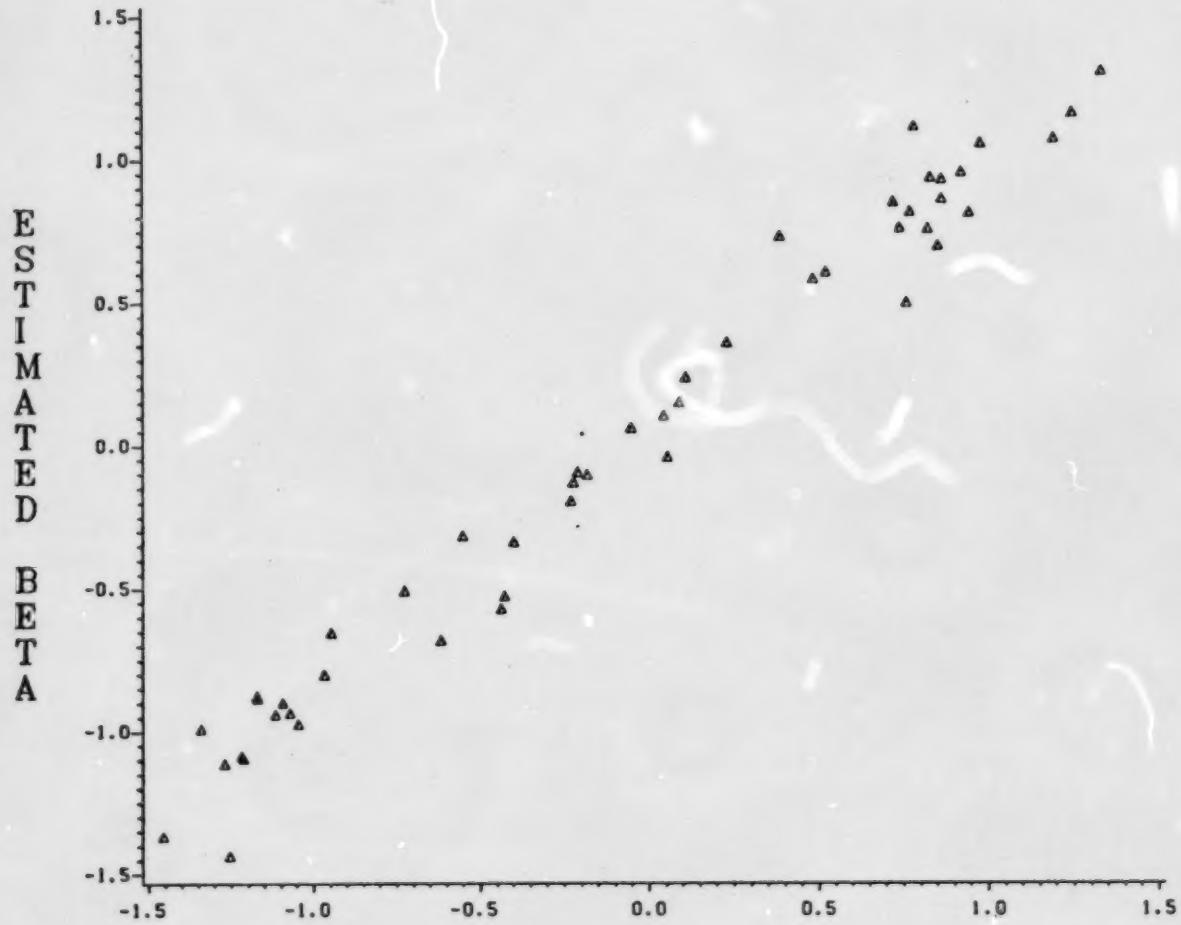

FIGURE 3

ESTIMATED AND ACTUAL THETAS
FROM SIMULATION WITH
N=200 AND K=50

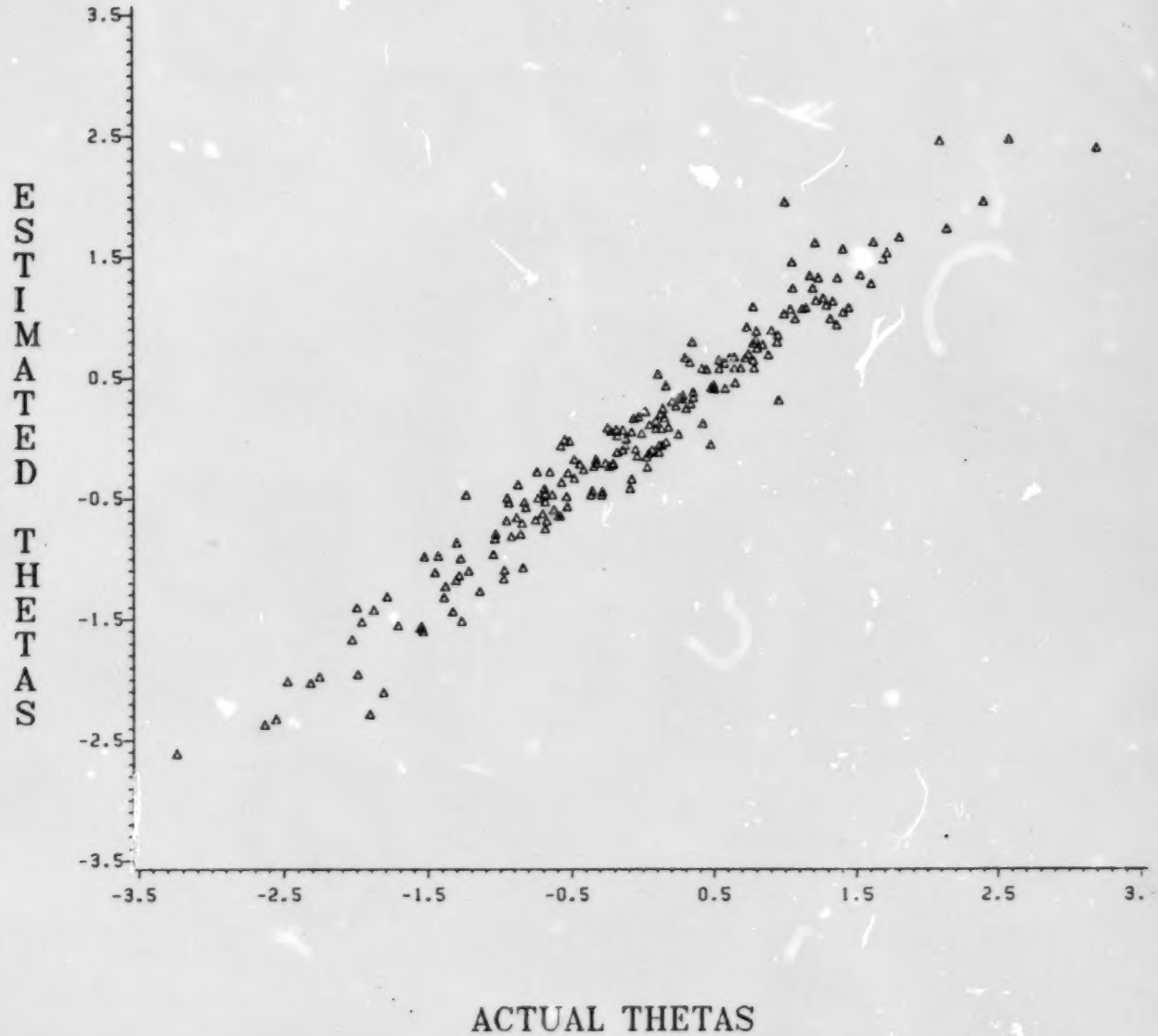

TABLE I

PARAMETER ESTIMATES AND COVARIANCES

FOR ARTHRITIS EXAMPLE

Item	$\hat{\alpha}$	$\hat{\beta}$	$\mathrm{Var}(\hat{\alpha})$	$\mathrm{Var}(\hat{\alpha},\hat{\beta})$	$\mathrm{Var}(\hat{\beta})$
1	1.14	-2.44	0.1287	0.1910	0.3490
2	1.96	-1.69	0.1837	0.0381	0.0310
3	0.62	-3.96	0.0722	0.4022	2.5197
4	1.84	-1.64	0.1552	0.0365	0.0353
5	1.30	-2.03	0.1124	0.0988	0.1431
6	1.41	-1.86	0.0981	0.0508	0.0754
7	1.31	-1.85	0.0983	0.0700	0.1031
8	2.67	-1.28	0.3574	-0.0004	0.0111
9	1.20	-1.85	0.0765	0.0579	0.1033
10	2.12	-1.26	0.1855	0.0123	0.0192
11	1.12	-1.84	0.0715	0.0625	0.1205
12	0.73	-2.40	0.0436	0.1027	0.3838
13	1.05	-1.77	0.0562	0.0457	0.1053
14	0.95	-1.86	0.0538	0.0592	0.1467
15	0.65	-2.43	0.0418	0.1170	0.4909
16	1.65	-1.21	0.0983	0.0064	0.0253
17	0.70	-2.21	0.0491	0.1162	0.4076
18	0.36	-3.89	0.0370	0.3704	4.1652
19	0.69	-1.74	0.0431	0.0802	0.2588
20	0.52	-2.18	0.0400	0.1425	0.6913
21	1.52	-0.89	0.0728	-0.0030	0.0220
22	1.13	-0.98	0.0567	0.0151	0.0436
23	1.11	-0.96	0.0536	0.0150	0.0443
24	0.67	-1.36	0.0432	0.0643	0.1909
25	0.51	-1.45	0.0365	0.0871	0.3493
26	0.68	-0.98	0.0374	0.0363	0.1159
27	0.60	-0.91	0.0416	0.0523	0.1598
28	0.52	-1.05	0.0352	0.0593	0.2250
29	1.10	-0.39	0.0563	0.0058	0.0338
30	1.06	-0.40	0.0575	0.0085	0.0361
31	0.91	-0.33	0.0462	0.0081	0.0444
32	0.26	-1.06	0.0331	0.1350	0.9367
33	0.77	-0.17	0.0480	0.0074	0.0545
34	0.70	-0.19	0.0452	0.0094	0.0648
35	0.61	-0.18	0.0419	0.0121	0.0831
36	0.67	0.04	0.0478	-0.0003	0.0664
37	1.00	0.08	0.0649	-0.0034	0.0360
38	1.29	0.11	0.0954	-0.0035	0.0258
39	0.68	0.44	0.0506	-0.0242	0.0770
40	1.01	0.38	0.0758	-0.0176	0.0400
41	1.46	1.18	0.0466	-0.1052	0.3755
42	0.42	1.33	0.0442	-0.1222	0.4979
43	0.30	2.17	0.0444	-0.2974	2.3145
44	0.75	1.28	0.0712	-0.0952	0.1897
45	0.68	1.71	0.0734	-0.1503	0.3862
46	0.54	2.48	0.0686	-0.2752	1.2395
47	0.44	3.85	0.0840	-0.6664	5.5250

TABLE II

COVARIANCES AND STANDARDIZED ERRORS

FROM SIMULATION

Item	$\widehat{Var}(\alpha)$	$\widehat{Cov}(\alpha,\beta)$	$\widehat{Var}(\beta)$	Standardized Error $\hat{\alpha}$	Standardized Error $\hat{\beta}$
1	0.0913	0.0303	0.0320	0.33	0.52
2	0.0492	0.0220	0.0383	-0.14	1.78
3	0.1395	0.0200	0.0164	0.75	1.23
4	0.1042	0.0369	0.0347	-0.48	-0.96
5	0.3488	0.0178	0.0088	1.59	1.42
6	0.4080	0.0176	0.0082	1.42	1.29
7	0.1349	0.0100	0.0121	0.91	2.74
8	0.1032	0.0115	0.0151	0.75	2.33
9	0.1747	0.0107	0.0108	0.77	1.72
10	0.0581	0.0153	0.0268	0.24	1.19
11	0.0515	0.0185	0.0333	-1.31	0.77
12	0.0714	0.0167	0.0237	-0.28	0.48
13	0.1907	0.0057	0.0089	0.49	1.79
14	0.0746	0.0055	0.0160	0.57	2.32
15	0.1534	-0.0029	0.0086	-0.35	2.35
16	0.0496	0.0092	0.0255	0.24	-0.41
17	0.3044	-0.0112	0.0064	1.87	2.93
18	0.0561	0.0051	0.0202	0.05	-0.94
19	0.0855	0.0215	0.0166	-0.70	-0.78
20	0.0572	-0.0009	0.0178	-0.07	0.47
21	0.0589	-0.0052	0.0170	-0.77	0.28
22	0.0658	-0.0064	0.0155	1.01	0.75
23	0.0666	-0.0076	0.0155	0.96	0.94
24	0.0519	-0.0057	0.0195	0.71	0.57
25	0.1745	-0.0192	0.0101	1.11	1.12
26	0.0787	-0.0138	0.0150	0.03	0.49
27	0.0587	-0.0085	0.0176	0.34	-0.72
28	0.0905	-0.0161	0.0143	0.93	0.50
29	0.0390	-0.0129	0.0320	-1.39	0.70
30	0.0727	-0.0225	0.0195	0.55	0.92
31	0.0443	-0.0288	0.0440	-1.35	1.65
32	0.0495	-0.0251	0.0330	-0.98	0.55
33	0.1206	-0.0341	0.0197	-0.01	0.60
34	0.1315	-0.0464	0.0258	-0.12	0.81
35	0.0687	-0.0346	0.0316	0.52	0.11
36	0.0568	-0.0216	0.0261	0.19	-1.62
37	0.0781	-0.0400	0.0321	0.03	0.26
38	0.0716	-0.0520	0.0510	-0.89	1.47
39	0.0974	-0.0392	0.0261	0.25	-0.40
40	0.0647	-0.0433	0.0426	0.11	0.50
41	0.1214	-0.0403	0.0222	0.69	-1.04
42	0.0794	-0.0460	0.0370	0.50	0.35
43	0.1112	-0.0482	0.0287	0.58	-0.01
44	0.3021	-0.0824	0.0249	0.95	0.19
45	0.1651	-0.0559	0.0238	1.23	-0.85
46	0.0522	-0.0511	0.0533	-0.42	0.33
47	0.0909	-0.0608	0.0442	1.25	-0.57
48	0.0539	-0.0566	0.0714	-0.63	-0.31
49	0.1972	-0.1049	0.0475	1.45	-0.12
50	0.1587	-0.1260	0.0814	0.24	0.52

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Navy

1 Dr. Nick Bond
Office of Naval Research
Liaison Office, Far East
APO San Francisco, CA 96503

1 Lt. Alexander Bory
Applied Psychology
Measurement Division
NAMRL
NAS Pensacola, FL 32508

1 Dr. Robert Breaux
NAVTRAEQUIPCEN
Code N-095R
Orlando, FL 32813

1 Dr. Robert Carroll
NAVOP 115
Washington , DC 20370

1 Chief of Naval Education and Training
Liason Office
Air Force Human Resource Laboratory
Operations Training Division
WILLIAMS AFB, AZ 85224

1 Dr. Stanley Collyer
Office of Naval Technology
800 N. Quincy Street
Arlington, VA 22217

1 CDR Mike Curran
Office of Naval Research
800 N. Quincy St.
Code 270
Arlington, VA 22217

1 Mike Durmeyer
Instructional Program Development
Building 90
NET-PDCD
Great Lakes NTC, IL 60088

1 Dr. Richard Elster
Department of Administrative Sciences
Naval Postgraduate School
Monterey, CA 93940

1 DR. PAT FEDERICO
Code P13
NPRDC
San Diego, CA 92152

Navy

1 Dr. Cathy Fernandes
Navy Personnel R&D Center
San Diego, CA 92152

1 Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Jim Hollan
Code 14
Navy Personnel R & D Center
San Diego, CA 92152

1 Dr. Ed Hutchins
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Norman J. Kerr
Chief of Naval Technical Training
Naval Air Station Memphis (75)
Millington, TN 38054

1 Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. William L. Maloy (02)
Chief of Naval Education and Training
Naval Air Station
Pensacola, FL 32508

1 Dr. James McBride
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr William Montague
NPRDC Code 13
San Diego, CA 92152

1 Dr. William E. Nordbrock
FMC-ADCO Box 25
APO, NY 09710

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San Diego, CA 92152

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Navy Personnel R&D Center
San Diego, CA 92152

Navy

6 Personnel & Training Research Group
Code 442PT
Office of Naval Research
Arlington, VA 22217

1 Special Asst. for Education and
Training (OP-01E)
Rm. 2705 Arlington Annex
Washington, DC 20370

1 Office of the Chief of Naval Operations
Research Development & Studies Branch
OP 115
Washington, DC 20350

1 LT Frank C. Petho, MSC, USN (Ph.D)
CNET (N-432)
NAS
Pensacola, FL 32508

1 Dr. Bernard Rimland (01C)
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

1 Dr. Robert G. Smith
Office of Chief of Naval Operations
OP-987H
Washington, DC 20350

1 Dr. Alfred F. Smode, Director
Training Analysis & Evaluation Group
Dept. of the Navy
Orlando, FL 32813

1 Dr. Richard Snow
Liaison Scientist
Office of Naval Research
Branch Office, London
Box 39
FPO New York, NY 09510

1 Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Frederick Steinheiser
CNO - OP115
Navy Annex
Arlington, VA 20370

Navy

1 Mr. Brad Sympson
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Martin A. Tolcott
Leader, Psychological Sciences Division
Office of Naval Research
800 N. Quincy St.
Arlinsgon VA 22217

1 Dr. James Tweeddale
Technical Director
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Frank Vicino
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Edward Wegman
Office of Naval Research (Code 411S&P)
800 North Quincy Street
Arlington, VA 22217

1 Dr. Ronald Weitzman
Naval Postgraduate School
Department of Administrative
Sciences
Monterey, CA 93940

1 Dr. Douglas Wetzel
Code 12
Navy Personnel R&D Center
San Diego, CA 92152

1 DR. MARTIN F. WISKOFF
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

1 Mr John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
San Diego, CA 92152

Marine Corps

1 H. William Greenup
Education Advisor (E031)
Education Center, MCDEC
Quantico, VA 22134

1 Director, Office of Manpower Utilizatio
HQ, Marine Corps (MPU)
BCB, Bldg. 2009
Quantico, VA 22134

1 Headquarters, U. S. Marine Corps
Code MPI-20
Washington, DC 20380

1 Special Assistant for Marine
Corps Matters
Code 100M
Office of Naval Research
800 N. Quincy St.
Arlington, VA 22217

1 DR. A.L. SLAFKOSKY
SCIENTIFIC ADVISOR (CODE RD-1)
HQ, U.S. MARINE CORPS
WASHINGTON, DC 20380

1 Major Frank Yohannan, USMC
Headquarters, Marine Corps
(Code MPI-20)
Washington, DC 20380

Army

1 Technical Director
U. S. Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Mr. James Baker
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Blvd.
Alexandria , VA 22333

1 Dr. Beatrice J. Farr
U. S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Myron Fischl
U.S. Army Research Institute for the
Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Milton S. Katz
Training Technical Area
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Harold F. O'Neil, Jr.
Director, Training Research Lab
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Commander, U.S. Army Research Institute
for the Behavioral & Social Sciences
ATTN: PERI-BR (Dr. Judith Orasanu)
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Joseph Psotka, Ph.D.
ATTN: PERI-1C
Army Research Institute
5001 Eisenhower Ave.
Alexandria, VA 22333

Army

1 Mr. Robert Ross
U.S. Army Research Institute for the
Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Robert Sasmor
U. S. Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 DR. ROBERT J. SEIDEL
US Army Research Institute
5001 Eisenhower Ave.
300 N. WASHINGTON ST.
Alexandria, VA 22333

1 Dr. Joyce Shields
Army Research Institute for the
Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Hilda Wing
Army Research Institute
5001 Eisenhower Ave.
Alexandria, VA 22333

Air Force

1 Air Force Human Resources Lab
AFHRL/MPD
Brooks AFB, TX 78235

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Air Force Human Resources Laboratory
WPAFB, OH 45433

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Research
Life Sciences Directorate, NL
Bolling Air Force Base
Washington, DC 20332

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Maxwell AFB, AL 36112

1 Dr. Earl A. Alluisi
HQ, AFHRL (AFSC)
Brooks AFB, TX 78235

1 Mr. Raymond E. Christal
AFHRL/MOE
Brooks AFB, TX 78235

1 Dr. Alfred R. Fregly
AFOSR/NL
Bolling AFB, DC 20332

1 Dr. Genevieve Haddad
Program Manager
Life Sciences Directorate
AFOSR
Bolling AFB, DC 20332

1 Dr. T. M. Longridge
AFHRL/OTE
Williams AFB, AZ 85224

1 Mr. Randolph Park
AFHRL/MOAN
Brooks AFB, TX 78235

1 Dr. Roger Pennell
Air Force Human Resources Laboratory
Lowry AFB, CO 80230

1 Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

Air Force

1 3700 TCHTW/TTGHR
2Lt Tallarigo
Sheppard AFB, TX 76311

1 Lt. Col James E. Watson
HQ USAF/MPXOA
The Pentasgon
Washington, DC 20330

1 Major John Welsh
AFHRL/MOAN
Brooks AFB , TX

1 Dr. Joseph Yasatuke
AFHRL/LRT
Lowry AFB, CO 80230

Department of Defense

12 Defense Technical Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC

1 Dr. Craig I. Fields
Advanced Research Projects Agency
1400 Wilson Blvd.
Arlington, VA 22209

1 Dr. William Graham
Testing Directorate
MEPCOM/MEPCT-P
Ft. Sheridan, IL 60037

1 Jerry Lehnus
HQ MEPCOM
Attn: MEPCT-P
Fort Sheridan, IL 60037

1 Military Assistant for Training and
Personnel Technology
Office of the Under Secretary of Defens
for Research & Engineering
Room 3D129, The Pentagon
Washington, DC 20301

1 Dr. Wayne Sellman
Office of the Assistant Secretary
of Defense (MRA & L)
2B269 The Pentagon
Washington, DC 20301

1 Major Jack Thorpe
DARPA
1400 Wilson Blvd.
Arlington, VA 22209

1 Dr. Robert A. Wisher
OUSDRE (ELS)
The Pentagon, Room 3D129
Washington, DC 20301

Civilian Agencies

1 Dr. Susan Chipman
Learning and Development
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Arthur Melmed
724 Brown
U. S. Dept. of Education
Washington, DC 20208

1 Dr. Andrew R. Molnar
Office of Scientific and Engineering
Personnel and Education
National Science Foundation
Washington, DC 20550

1 Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E Street NW
Washington, DC 20415

1 Mr. Thomas A. Warm
U. S. Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

1 Dr. Joseph L. Young, Director
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Private Sector

1 Dr. James Algina
University of Florida
Gainesville, FL 326

1 Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

1 Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

1 Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Tel Aviv, Ramat Aviv 69978
Israel

1 Dr. Werner Birke
Personalstammamt der Bundeswehr
D-5000 Koeln 90
WEST GERMANY

1 Dr. R. Darrell Bock
Department of Education
University of Chicago
Chicago, IL 60637

1 Dr. Robert Brennan
American College Testing Programs
P. O. Box 168
Iowa City, IA 52243

1 Dr. Glenn Bryan
6208 Poe Road
Bethesda, MD 20817

1 Dr. Ernest R. Cadotte
307 Stokely
University of Tennessee
Knoxville, TN 37916

1 Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

1 Dr. Norman Cliff
Dept. of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

Private Sector

1 Dr. Allan M. Collins
Bolt Beranek & Newman, Inc.
50 Moulton Street
Cambridge, MA 02138

1 Dr. Hans Crombag
Education Research Center
University of Leyden
Boerhaavelaan 2
2334 EN Leyden
The NETHERLANDS

1 CTB/McGraw-Hill Library
2500 Garden Road
Monterey, CA 93940

1 Dr. Dattpradad Divgi
Syracuse University
Department of Psychology
Syracuse, NE 33210

1 Dr. Fritz Drasgow
Department of Psychology
University of Illinois
603 E. Daniel St.
Champaign, IL 61820

1 Dr. Susan Embertson
PSYCHOLOGY DEPARTMENT
UNIVERSITY OF KANSAS
Lawrence, KS 66045

1 ERIC Facility-Acquisitions
4833 Rugby Avenue
Bethesda, MD 20014

1 Dr. Benjamin A. Fairbank, Jr.
McFann-Gray & Associates, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

1 Dr. Leonard Feldt
Lindquist Center for Measurment
University of Iowa
Iowa City, IA 52242

1 Dr. Richard L. Ferguson
The American College Testing Program
P.O. Box 168
Iowa City, IA 52240

Private Sector

1 Univ. Prof. Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

1 Professor Donald Fitzgerald
University of New England
Armidale, New South Wales 2351
AUSTRALIA

1 Dr. Dexter Fletcher
University of Oregon
Department of Computer Science
Eugene, OR 97403

1 Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138

1 Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01002

1 Dr. Robert Glaser
Learning Research & Development Center
University of Pittsburgh
3939 O'Hara Street
PITTSBURGH, PA 15260

1 Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

1 Dr. Ron Hambleton
School of Education
University of Massachusetts
Amherst, MA 01002

1 Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 90010

1 Dr. Lloyd Humphreys
Department of Psychology
University of Illinois
603 East Daniel Street
Champaign, IL 61820

Private Sector

1 Dr. Earl Hunt
Dept. of Psychology
University of Washington
Seattle, WA 98105

1 Dr. Jack Hunter
2122 Coolidge St.
Lansing, MI 48906

1 Dr. Huynh Huynh
College of Education
University of South Carolina
Columbia, SC 29208

1 Dr. Douglas H. Jones
Advanced Statistical Technologies
Corporation
10 Trafalgar Court
Lawrenceville, NJ 08148

1 Professor John A. Keats
Department of Psychology
The University of Newcastle
N.S.W. 2308
AUSTRALIA

1 Dr. William Koch
University of Texas-Austin
Measurement and Evaluation Center
Austin, TX 78703

1 Dr. Marcy Lansman
The L. L. Thurstone Psychometric
Laboratory
University of North Carolina
Davie Hall 013A
Chapel Hill, NC 27514

1 Dr. Alan Lesgold
Learning R&D Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

1 Dr. Michael Levine
Department of Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Private Sector

1 Dr. Charles Lewis
Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Oude Boteringestraat 23
9712GC Groningen
Netherlands

1 Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

1 Mr. Phillip Livingston
Systems and Applied Sciences Corporatio
6811 Kenilworth Avenue
Riverdale, MD 20840

1 Dr. Robert Lockman
Center for Naval Analysis
200 North Beauregard St.
Alexandria, VA 22311

1 Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

1 Dr. James Lumsden
Department of Psychology
University of Western Australia
Nedlands W.A. 6009
AUSTRALIA

1 Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

1 Dr. Scott Maxwell
Department of Psychology
University of Notre Dame
Notre Dame, IN 46556

1 Dr. Samuel T. Mayo
Loyola University of Chicago
820 North Michigan Avenue
Chicago, IL 60611

1 Mr. Robert McKinley
American College Testing Programs
P.O. Box 168
Iowa City, IA 52243

Private Sector

1 Dr. Barbara Means
Human Resources Research Organization
300 North Washington
Alexandria, VA 22314

1 Dr. Robert Mislevy
711 Illinois Street
Geneva, IL 60134

1 Dr. Allen Munro
Behavioral Technology Laboratories
1845 Elena Ave.. Fourth Floor
Redondo Beach, CA 90277

1 Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

1 Dr. Donald A Norman
Cognitive Science, C-015
Univ. of California, San Diego
La Jolla, CA 92093

1 Dr. Melvin R. Novick
356 Lindquist Center for Measurment
University of Iowa
Iowa City, IA 52242

1 Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

1 Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Cirle, NW
Washington, DC 20036

1 Dr. James A. Paulson
Portland State University
P.O. Box 751
Portland, OR 97207

1 Dr. James W. Pellegrino
University of California,
Santa Barbara
Dept. of Psychology
Santa Barabara, CA 93106

Private Sector

1 Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

1 Dr. Lauren Resnick
LRDC
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 1521

1 Dr. Thomas Reynolds
University of Texas-Dallas
Marketing Department
P. O. Box 688
Richardson, TX 75080

1 Dr. Andrew M. Rose
American Institutes for Research
1055 Thomas Jefferson St. NW
Washington, DC 20007

1 Dr. Lawrence Rudner
403 Elm Avenue
Takoma Park, MD 20012

1 Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

1 PROF. FUMIKO SAMEJIMA
DEPT. OF PSYCHOLOGY
UNIVERSITY OF TENNESSEE
KNOXVILLE, TN 37916

1 Dr. Walter Schneider
Psychology Department
603 E. Daniel
Champaign, IL 61820

1 Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

1 Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujusawa 251
JAPAN

Private Sector

1 Dr. Edwin Shirkey
Department of Psychology
University of Central Florida
Orlando, FL 32816

1 Dr. William Sims
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

1 Dr. H. Wallace Sinaiko
Program Director
Manpower Research and Advisory Services
Smithsonian Institution
801 North Pitt Street
Alexandria, VA 22314

1 Dr. Kathryn T. Spoehr
Psychology Department
Brown University
Providence, RI 02912

1 Dr. Robert Sternberg
Dept. of Psychology
Yale University
Box 11A, Yale Station
New Haven, CT 06520

1 Dr. Peter Stoloff
Center for Naval Analysis
200 North Beauregard Street
Alexandria, VA 22311

1 Dr. William Stout
University of Illinois
Department of Mathematics
Urbana, IL 61801

1 DR. PATRICK SUPPES
INSTITUTE FOR MATHEMATICAL STUDIES IN
THE SOCIAL SCIENCES
STANFORD UNIVERSITY
STANFORD, CA 94305

1 Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Private Sector

1 Dr. Kikumi Tatsuoka
Computer Based Education Research Lab
252 Engineering Research Laboratory
Urbana, IL 61801

1 Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

1 Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

1 Dr. Douglas Towne
Univ. of So. California
Behavioral Technology Labs
1845 S. Elena Ave.
Redondo Beach, CA 90277

1 Dr. Robert Tsutakawa
Department of Statistics
University of Missouri
Columbia, MO 65201

1 Dr. J. Uhlaner
Uhlaner Consultants
4258 Bonavita Drive
Encino, CA 91436

1 Dr. V. R. R. Uppuluri
Union Carbide Corporation
Nuclear Division
P. O. Box Y
Oak Ridge, TN 37830

1 Dr. David Vale
Assessment Systems Corporation
2233 University Avenue
Suite 310
St. Paul, MN 55114

1 Dr. Kurt Van Lehn
Xerox PARC
3333 Coyote Hill Road
Palo Alto, CA 94304

1 Dr. Howard Wainer
Division of Psychological Studies
Educational Testing Service
Princeton, NJ 08540

Private Sector

1 Dr. Michael T. Waller
Department of Educational Psychology
University of Wisconsin--Milwaukee
Milwaukee, WI 53201

1 Dr. Brian Waters
HumRRO
300 North Washington
Alexandria, VA 22314

1 Dr. Phyllis Weaver
2979 Alexis Drive
Palo Alto, CA 94304

1 Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

1 Dr. Donald O. Weitzman
Mitre Corporation
1820 Dolley Madison Blvd
McLean, VA 22102

1 Dr. Rand R. Wilcox
University of Southern California
Department of Psychology
Los Angeles, CA 90007

1 Wolfgang Wildgrube
Streitkraefteamt
Box 20 50 03
D-5300 Bonn 2
WEST GERMANY

1 Dr. Bruce Williams
Department of Educational Psychology
University of Illinois
Urbana, IL 61801

1 Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940